Newell’s method

Goal: Need to find the normal for a possibly non-planar polygon.

Let’s look at a quadrilateral as an example. Divide the quadrilateral into two triangles.

A close up of a map

Description automatically generated

Find the normal for each triangle and add them together (this gives a scaled version of the average normal).

The normal for the triangle made up of P0, P1, and P2 is,

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For the x component of the normal, the value is (*y*1-*y*0)(*z*2-*z*0) - (*y*2-*y*0)(*z*1-*z*0).

Similarly, for the triangle made up of P0, P2, and P3, the *x* component of the normal is (*y*2-*y*0)(*z*3-*z*0) - (*y*3-*y*0)(*z*2-*z*0).

The total normal is the sum of the previous two expressions:

(*y*1-*y*0)(*z*2-*z*0) - (*y*2-*y*0)(*z*1-*z*0) +

(*y*2-*y*0)(*z*3-*z*0) - (*y*3-*y*0)(*z*2-*z*0) =

*y*1*z*2-*y*1*z*0-*y*2*z*1+*y*0*z*1+*y*2*z*3-*y*0*z*3-*y*3*z*2+*y*3*z*0

Add and subtract (giving a zero-sum change) terms of *y*0*z*0, *y*1*z*1, *y*2*z*2, *y*3*z*3:

*y*1*z*2-*y*1*z*0-*y*2*z*1+*y*0*z*1+*y*2*z*3-*y*0*z*3-*y*3*z*2+ *y*3*z*0+*y*0*z*0-*y*0*z*0+*y*1*z*1-*y*1*z*1+*y*2*z*2-*y*2*z*2+ *y*3*z*3 -*y*3*z*3

Rearrange terms for grouping:

*y*0*z*0+*y*0*z*1-*y*1*z*0-*y*1*z*1 +

*y*1*z*1+*y*1*z*2-*y*2*z*1-*y*2*z*2 +

*y*2*z*2+*y*2*z*3-*y*3*z*2-*y*3*z*3 +

*y*3*z*3+*y*3*z*0-*y*0*z*3-*y*0*z*0

Grouping the previous expression in the “right” way gives the normal as

(*y*0-*y*1)(*z*0+*z*1) *+* (*y*1-*y*2)(*z*1+*z*2) + (*y*2-*y*3)(*z*2+*z*3) + (*y*3-*y*0)(*z*3+*z*0)

This expression can be generalized to

Σ(*y*i-*y*j)(*z*i+*z*j) where j=(i+1)%n.

There are similar expressions for the *y* component (Σ(*z*i-*z*j)(*x*i+*x*j)) and the *z* component (Σ(*x*i-*x*j)(*y*i+*y*j)) of the normal.